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2019

22nd Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 10057

Problem Chosen: A

Our paper aims to identify and predict trends of free energy consumption from 2011-2021, model the implicit and monetary costs of free charging, analyze the costs from multiple perspectives, and optimize costs to maximize societal welfare. We split the questions into two categories: electrical vehicles (EVs) charging stations and devices (phones and laptops) charging outlets, and considered data from four regions: Global, China, United States, and Europe.

In Q1, we deduced that the total free energy consumed by EVs is a function of the number of EVs, the mileage per year, and the energy per mile. Similarly, for devices, the total is an aggregate of the number of devices, time use per year, and energy per unit time. We first collected data on the sales of EVs and devices, using Matlab to apply regression. Then, we considered the wear-off, or lifetime of an EV/device, utilizing a moving integral to obtain the operating number of EVs and devices in any year. After that we fitted our data with a suitable rates of energy use to determine a general function for energy consumption. Then, we used Keynes' Law that states demand creates its own supply to prove the positive correlation between the total free energy consumption and number of free charging stations for both devices and EVs. Our results quantitatively showed that free energy sources are projected to increase into 2021 for all regions in general, with this result leading to large impacts on public places that fulfill a certain set of requirements.

For Q2, we broke down the costs for EVs and devices into four components: monetary cost (measurable fixed and variable costs), space cost (opportunity cost and space hogging), security cost (juice jacking and theft), and convenience benefits (negative costs; positive externalities of the ports). To do so, we analyzed a wealth of data from online sources, combining this exterior data with our own personal knowledge to derive quantitative relationships that generally apply to all places, considering in detail how these costs are shared over the different stakeholders. This includes exponential charging functions and Cobb-Douglas econometrics functions that allow us to accurately model the cost relationship. For Q3, we considered consumer demand, location of ports, and level of security to determine how each component influences the Cost Model when it is moved into a different area. Then, for Q4, we used Lagrange optimization to perform multivariable minimization on the Cost Function by dividing it into four functions representing the four smaller costs and determining restrictive relationships between the variables. We calculated the partial derivatives and combined the equations to determine the multiple Lagrange multipliers, determining the saddle points that minimize the cost function for the entire model.

As a whole, our model has a strong foundational research that synthesizes over 70 web sources with complex methodology and a high level of generalizability through consideration of both regional and global data and realistic assumptions. Our results show a high degree of prediction, showing that we can specifically, explicitly determine how to optimize our energy uses.

Table of Contents

Newspaper Release

Charge: The Secret Costs Behind Free Energy

Written by: Team 10057 | 19 November 2019

Have you ever charged your phone in a public outlet or brought your electrical vehicle to a free charging station? If so, you have already had first hand experience with the increasingly popular trend of “free” energy consumption. This type of energy consumption is offered in both private and government owned institutions such as cafés, schools, and airports.

Our team tracked the development of these “free” sources of energy from 2011-2018 globally and across China, Europe, and the United States. We divided the sources into two key categories: electrical vehicles and mobile devices, and found an overall increasing trend for both types. Then, we predicted future developments from 2019-2021. As a whole, free energy consumption is projected to continue increasing in the short term, with EVs growing more rapidly than devices and China in the lead.

The concept of ever-increasing free energy sources seems nearly too good to be true; but is this really free? After in depth analysis of both implicit and explicit costs during the creation of these charging stations, our team came up with a list of four significant components: monetary costs, space costs, security costs, and convenience benefits.

Yet, these costs do not mean “free” energy sources are a lost cause. Charging stations for EVs and outlets for mobile devices are both essential for the success of businesses as well as environment protection. Electrical vehicles, especially, are more beneficial for the environment because they have zero exhaust emissions, use renewable energy, and are majorly made up of eco-friendly materials [49]. With the current global crises of climate change and environmental degradation, this increasing trend of EV charging stations is a significant step to reducing pollution and maintaining a healthier relationship with the Earth.

However, even though free energy sources are becoming more ubiquitous, that does not mean one should take advantage of these public services by hogging the mobile device chargers in cafés or stealing charging phones in public airports. These actions will only expand the space and security costs for firms providing these services, making it less likely that this trend of free energy will continue increasing in the future. The concept of “free” energy is a misnomer as there are definitely implicit and explicit costs to consider.

With this in mind, our team set up a model to minimize the costs and develop methods to optimize societal welfare. Using a strategy called the Lagrange Multiplier, we preformed multivariable optimization with partial derivatives. Using a series of these multipliers and derivatives, we obtained the final optimal point that maximizes benefit and minimizes the cost, so that you all can freely enjoy the conveniences that this free energy brings!

Therefore, with the joint cooperation of the communities surrounding these charging stations, we can, and we have optimized these costs to provide a better future for everyone to share the fruits of technology.

1. INTRODUCTION

*All citations throughout the paper are done with [x]; x refers to the number in the bibliography (see Appendix A.1); definitions are recorded in footnotes; due to page limits, much of the intermediate data is stored in Appendix B.

1.1 Background

With the onset of the digital age, technology has developed rapidly, leading to the proliferated use of electronic devices, particularly mobile devices and electric vehicles. This trend is matched by a similar expansion in global public charging sources in forms such as electrical outlets and charging stations. A majority of these public sources of electricity provide their services for free or a very low price. Yet, while this concept seems beneficial to social welfare, there still remains a number of costs and consequences of these "free" charging sites. In this paper, we intend to discover and quantify the costs, and then propose a possible way of minimizing the costs in a number of scenarios.

1.2 Restatement of Problem

- Q1: Determine the trend of "free" energy consumption over recent years and to predict future patterns of usage and identify impacts of this trend on public places, and the requirements that must be met for places to install these public charging stations.
- Q2: Develop a model that analyzes the previously identified costs and how they are paid.
- Q3: Apply the model to various scenarios (i.e.: different public places)
- Q4: Determining methods to minimize cost and using them to adjust the cost model

1.3 Assumptions & Justifications

Table 1: Assumptions and Justifications

Assumptions	Justifications
The data for average per-mile energy consumption is accurate and applicable to our model.	For model simplification, we assume the data collected is accurate. Even if it is possible to create a weighted average per-mile energy consumption, the process will be unnecessarily complicated.
All EVs that are sold in the market are used to their maximum potential.	Without this assumption, there will be unquantifiable discrepancies in data that make our model unnecessarily complicated.
There is a positive correlation between the amount of energy consumed and the supply of charging stations and device outlets.	As there is limited data on the actual amount of charging stations and especially mobile device chargers, we used Keynes' Law that demand creates supply to estimate the number.
Energy / mile is constant for all calculations related to electric vehicles globally, and in China, US, and Europe.	These constants are assumed for model simplification, as the values are relatively similar across regions and it would be too complex to take all the models of each EV and mobile device into consideration.
Unit electricity cost per kWh is constant for all EVs and mobile devices.	
Wear-off ratio[1] is constant world wide.	
Annual mileage of EVs is constant and equals to mileage driven on a normal gasoline car.	There is insufficient data on mileage driven by EVs, so we used the closest alternative. EVs are more cost-efficient[2], which encourages people to drive more; but range anxiety[3] acts as a counter-factor. Hence, it is reasonable to assume that EV mileage is equal to that of a regular gasoline car.

[1] Amount of time an electric vehicle or mobile device can be used before having to be replaced
[2] eGallons are $1.43 cheaper on average than regular gasoline [1]
[3] The fear that EVs can not go far enough on a single charge [2]

2. Question One: Change & Predictions

2.1 Defining the Terms

We first divided free energy consumption into two categories: electrical vehicle (EV) charging stations and mobile devices (i.e.: phones and laptops) charging outlets because they are the two largest sources of "free" energy provided to the public. We defined "recent years" as the period from 2011-2018, because 2011 is the year when EVs emerged in global markets at a relatively lower price [3], and a majority of the accessible data online for both categories is within this time range.

2.2 Change and Predictions for EVs

As we were faced with a lack of data around number of charging stations around the world, we applied the concept of Keynes' Law[4] that suggests demand creates its own supply[4]. In this scenario, demand refers to the total energy consumption by electric vehicles, and supply refers to the number of charging stations required to meet these demands. We are aware that not all energy consumption is free, but it does constitute 5% of all energy consumption in EVs [16]. Hence, we assume that there is a positive correlation between the amount of energy consumed (the amount of electric vehicles demanded) and the supply of charging stations (the source of free energy).

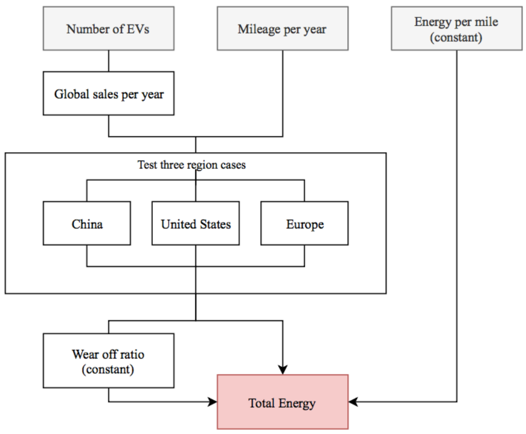


Figure 1: Breakdown of EV Energy Consumption

Figure 1 illustrates a breakdown of our process to find the energy consumption. First, we found the total number of EVs globally and per region (China, Europe, and the United States[5]) and determine the wear-off ratio[6]. Second, we found the mileage per year of the average car globally and regionally. Third, we found the energy used per mile when driving a car. Then, we combined the three factors together with the formula: $E(n) = n_{EV} \times m_{EV} \times E_{mile}$ where $E(n)$ refers to the total energy consumption, n_{EV} refers to the total number of electrical vehicles per year, m_{EV} refers to the average mileage per year[7], and E_{mile} refers to the amount of energy expended per mile.

Number of EVs—To obtain the data on number of EVs, we found statistics on the EV sales globally and regionally (China, Europe, US) (see Appendix B.2.5) [6][7][8][9]. The wear off ratio is assumed to be constant world-wide, and is set to be 8 years, which is the length of battery coverage [10]. Our analysis of the datasets to get the number of EVs will be discussed later in the paper.

Mileage Per Year—We set the annual mileage to be constant in every region because the rate of change in vehicle miles does not change drastically across time in the same country[8]. From a

[4] From John Maynard Keynes's *The General Theory of Employment, Interest, and Money* (1936) [46]
[5] We chose these three regions because they are the largest EV markets [5], making them most relevant to our paper.
[6] The amount of years each EV can last before a new one is required.
[7] Average length driven per car, per year
[8] Evidence of this is in data of mileage in the United States from 2010 to 2017 (see Appendix B.1.1)

compilation of multiple sources, we concluded the per region and global annual mileages (see Appendix B.2.8).

Energy Per Mile—We kept energy per mile driven as a constant 18kWh/100km [15].

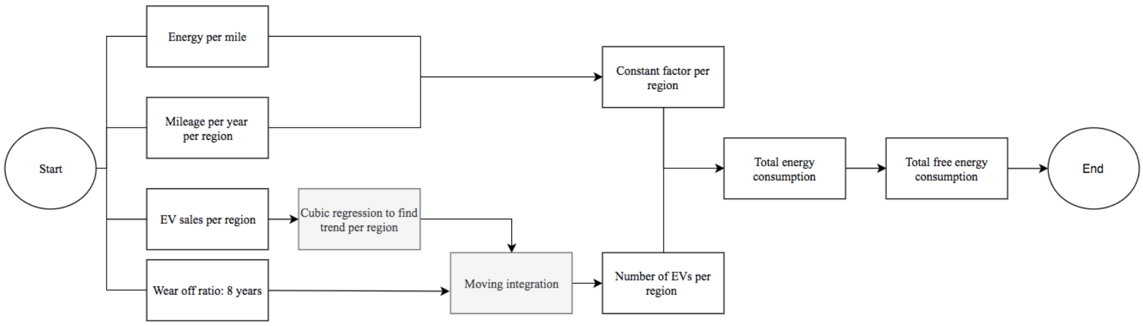


Figure 2: flow chart for finding free energy consumption trend per region, per year

Figure 2 shows our thought process for analyzing the raw data for each of the three factors[9] to get the final amount of free energy consumed per year[10] and per region[11]. First, we took the data of EV sales per region collected earlier and used a cubic regression to find the function for the trend of sales in each region, because with only 8 data points, a higher-degree polynomial will over-fit the curve; a lower-degree polynomial will under-fit the curve; and an exponential function is illogical because we cannot ensure that the rapid growth in sales will sustain over a long term period. In the regression model, we set the n-axis to be the difference between the current year and 2010[12]. As the curve must pass through the origin to make the function nonnegative, we assumed that EV production in recent years only begins in 2011. The regression function for global EV sales is shown below. (Regression formulas for the other three regions are in Appendix B.3.1.)

$$S(n) = 7.1795n^3 - 41.8630n^2 + 132.9368n$$

Then, we considered the wear off value of 8 years. We deduced that the total EVs available in the market will be a moving integral that crosses 8 units in x for $n > 8$. In other words, for $n < 8$, the available EVs will be the definite integral of the sales function between $x = 0$ and $x = n$, and for $n > 8$, the number will be the definite integral of the sales function between $x = n - 8$ and $x = n$. Summing the two separate functions, we obtain the final function for total number of EVs available:

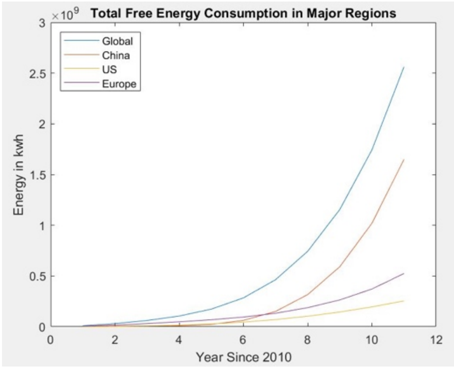
$$EV(n) = \int_{max(0,n-8)}^{n} S(n)\ dn$$

For our prediction model, we wanted to find the trend in from 2019-2021. Hence, we substitute 9, 10, and 11 for n in the equation for $E(n)$. (To see total number of EVs $E(n)$ from 2011-2021, see Appendix B.2.1). Then, we combined the result with the two other factors: mileage (per year per region) and energy per mile (constant). The factor is the product of average mileage driven in the region and per-mile energy consumption, which is set to be the same throughout. The formula for total energy consumed globally is: $E(n) = 11{,}500 \times 0.289682 \times n$, where $E(n)$ is the total energy, 0.289682 is the energy per mile; n is the number of years from 2010; 11,500 is the mileage. (To see the formulas for the other three regions, see Appendix B.3.2; To see the calculated data on total energy consumed from 2011-2021, see Appendix B.2.2.)

[9] Number of EVs, mileage per year, and energy per mile
[10] 2011-2018
[11] Globally, China, US, and Europe
[12] For example, in 2018, the n value will be 8; in 2012, the n value will be 2

After we got the total energy consumption, we calculated the free energy consumption $E_{PCS}(n)$ by multiplying 5% to $E(n)$, as 5% of all energy consumption is from public charging stations [16]. The final trends of free energy consumption per region from 2011-2018 and the predicted trends from 2019-2021 are illustrated in Figure 3 (for the processed data, see Appendix B.2.3) [5].

Figure 3: Total free energy consumption per region 2011-2021



2.3 Change and Prediction for Mobile Devices

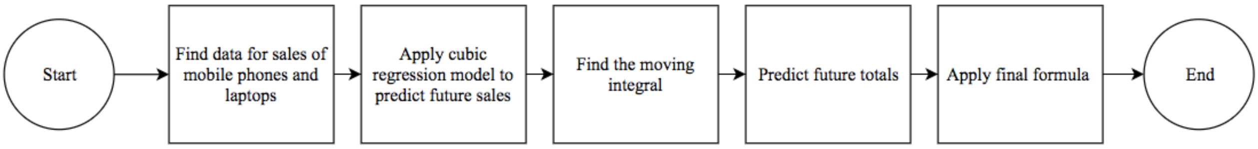
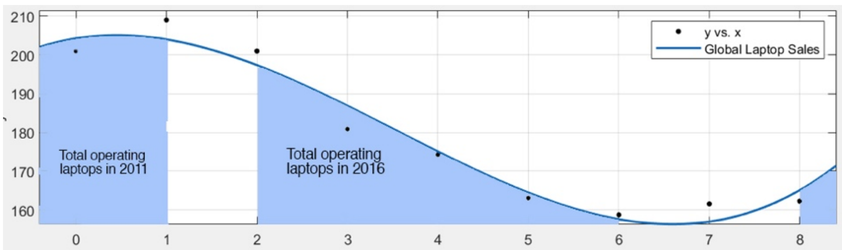


Figure 4: Breakdown of process for finding total free energy in devices

We divided mobile devices into two smaller subcategories: mobile phones and laptops, for they have the largest market share [17]. The figure shows the breakdown of our process for finding the total free energy use in mobile devices. First, we gathered the data of the total sales of laptops and phones from 2011-2018 [18]. Again, we used Keynes' Law and assumed the the demand of devices (illustrated through sales), would indicate the trend in supply of free energy (public charging ports). To predict future trends, we used a cubic regression with a relatively high R-squared value of 0.9624. The graph of global laptop sales is shown below, with the y-value (s(Y)) being global laptop sales per year in millions of years, and the x-value (n) being number of years after 2010.

Figure 5: Regression Model for Sale of Laptops 2011-2018



$$s(n_{laptops}) = 0.4199n^3 - 4.438n^2 + 3.744n + 204.3$$

$$s(n_{phones}) = 10.3284n^2 - 140.422n + 457.34$$

After determining the total sales per year, we applied a moving integral to calculate the total amount of operating laptops and phones in any year, because the change in total number of phones and laptops is equivalent to the total sales per year minus the number of laptops or phones that had already worn out and needed to be replaced. We had the total sales for each year, and a wear off ratio of 4 years for laptops and 2 years for phones [19]. The general and laptop/phone specific integral functions are as follows, n is year, w is wear out rate, and $s(n)$ is sales.

$$\int_{n-w}^{n} s(n)\,dn \qquad \int_{n-4}^{n} s(n_{laptops})\,dn \qquad \int_{n-2}^{n} s(n_{phones})\,dn$$

This will return the total amount of operating laptops and phones in the year being studied (listed in Appendix B.2.4.) Then, we used the equations to predict the total number of operating laptops and phones in future years, focusing on 2019-2021 to guarantee the applicability of our data (Data of laptop and phone sales from 2019-2021 are in Appendix B.2.9); To see additional data on the percent error of predicted sales, see Appendix B.2.7). Once these data values had been calculated, we used the formula $E_{total} = n_{Dev} \times hr \times E_{hr}$ to calculate total energy consumption, where n_{Dev} represents the number of devices, hr stands for how many hours an average person uses their device per day, and E_{hr} stands for energy use per hour. Inputting the average values found online, we calculated the two equations to be [20][21]:

$$E_{laptop} = n_{laptop} \times 4.117 \times 75 \qquad E_{phones} = n_{phones} \times 4.658 \times 5.5$$

Next, we summed the energy consumption per day and converted the data into the standard unit of kWh per year to align with the earlier data for EVs. We also multiplied the number by 0.05, assuming that the coefficient is the same for EVs and mobile devices. The final datasets are in Table 2.

Table 2: Predicted number of laptops and phones and energy consumption 2019-2021

Year	Operating Laptops (millions)	Daily E_{Laptop} (MWh)	Operating Phones (millions)	Daily E_{Phones}(MWh)	Daily E_{Dev} (MWh)	Annual E_{Dev} (kWh)	Annual E_{Dev_P}(kWh)
2019	651.646	201195	1428.77	36600.141	237796.141	86795226470	4339761324
2020	690.913	213319	1531.22	39224.403	252543.403	92430885500	4621544275
2021	775.297	239272	1667.20	42707.726	281979.726	102922600000	5146130000

Figure 6: Total Energy Consumption for Mobile Devices

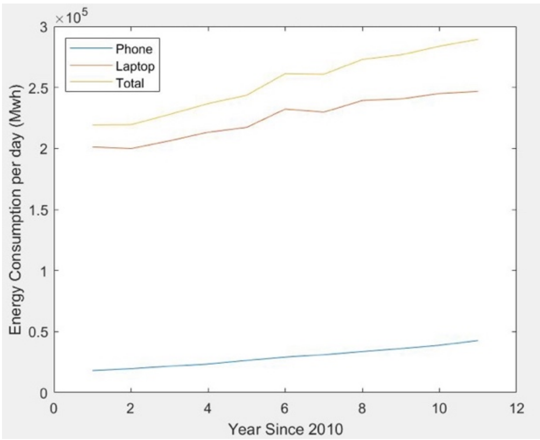


Figure 6 shows that total energy consumption is increasing per year, so by Keynes's Law, the total amount of sockets supplied will increase as well, meaning the number of free charging ports and free energy consumption has increased and will continue to do so in 2019-2021.

2.4 Additional Considerations

Besides the quantitative trends in energy consumption, there are a few other considerations that are less quantifiable but should still be considered.

2.4.1 Trend in consumer preferences: Phones vs Laptops—There is an increasing trend in both the total number of phones and laptops. This is in accordance with the larger global trend of increased consumer preferences towards mobile electronics [23]. In general, our model predicts that more charging stations will be built around the world, from urban metropolitans to the rapidly digitalizing suburbs.

2.4.2 Trend in consumer preferences: Electric Vehicles**—**There is an increased preference for EVs in comparison to regular gasoline cars. The global gasoline car numbers are expected to double once every 20 years at most, while our calculated number of EVs will triple every two years [24]. Therefore, EV charging stations are likely to replace gasoline stations in the near future.

2.4.3 Trend in public industries**—**Public, government-supported institutions (e.g.: airports, train stations) have demonstrated an increase in supply of free public charging. Airports such as London's Heathrow have dedicated much of their funds towards increasing the number of ports, resulting in an average of one charging port for every two passengers [25]. Hence, as consumer demands for EVCS rises, public industries are projected to increase their supply to meet them.

2.4.4 Trend in private industries**—**Yet, for private industries that have limited space usage (e.g.: coffee shops, restaurants), there is a decreasing trend of public free power consumption in device outlets. Many Starbucks shops in costly metropolitan area places such as Manhattan are blocking up many of their charging ports to discourage people who spend all their time hogging the space that could otherwise used for more customers [26]. However, shops that are decreasing their ports should also consider the greater benefits they bring, namely more customers attracted to the idea of free charging. Similarly, for EVs, CS that are placed in high-traffic environments like popular malls encourage users to visit these malls, hence increasing the revenue for the shops within. This cost-benefit analysis will be explored in more detail later on in the paper [27].

2.5 Requirements for Public Places

- *Physical Environment*—There must be a high level of traffic through the area, so each station and port is used to its maximum potential, and the power grid in the area must be able to support the increased demand for energy [47]. The location of these stations should also include a high population density of EV and device users that place great value on such stations so that an increase in EVCS or charging ports would lead to a greater quantity of users.
- *Social-political Environment*—Government policy can greatly influence the supply of charging stations, especially for EVs by creating regulations, subsidies, and taxes [48]. Though this is not directly quantifiable (and will not be used in our cost model), it is still worth considering.

2.6 Impacts on Public Places

The impacts, or costs, on public places with free energy consumption sources for EVs and devices can be divided into four types: monetary cost and implicit costs (i.e.: space cost, security cost, and convenience benefit [negative cost]).

- Monetary costs refer to the relatively direct, easily measurable financial costs that are separated into fixed costs (i.e.: installation cost for charging stations and outlets; maintenance costs) and variable costs (i.e.: per unit electricity cost).
- Space costs refer to the the opportunity cost of one person taking up the spot for free charging (i.e.: space hogging), hence depriving others of the chance to charge at that location. Security costs (i.e.: juice jacking and stolen vehicles for EV CS; stolen devices for mobile devices outlets) are the implicit cost of the risk to the owners of the EVs or devices when they choose to charge at these public areas. Finally, convenience benefits (or negative costs) are the benefits brought to the places with such sources through increased customers, popularity, and revenue. These costs will be quantified and explored in further detail in the following section.

3. Question 2: The Cost Model

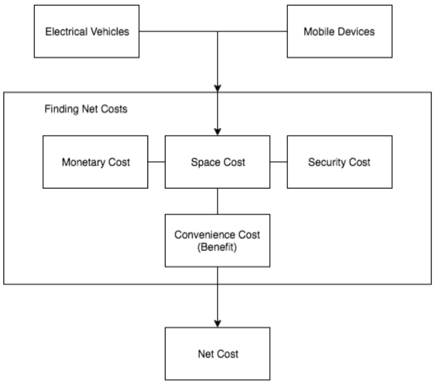


Figure 7: Breakdown of costs for EVs and Mobile Devices

3.1 Costs for EVs and How They Are Paid

3.1.1 Monetary costs

We divided monetary costs for EVs into fixed costs (TFC) and variable costs (TVC):

$$TC = TFC + TVC = \sum (C_{plug} \times \Delta n_{plug}) + C_{Elec} \times E_{EV}$$

C_{Plug} and C_{Elec} refer to the unit cost of electrical plugs and electricity respectively, while n_{Plug} represents the number of plugs added every year, and E_{EV} refers to the total energy consumption. To calculate TFC of one charging station, we first found the number of public charging stations globally. As there is a lack of data, we could only use the data of charging stations in the US from 2011-2018 and expand it to global number of charging stations [28]. We did this by using data of the number of EVs in the US (Q1) to get the ratio of electrical vehicles to charging stations in the US per year. After we got this ratio (generally increasing per year), we used regression to predict the ratio of future years from 2019-2021 (full data of EVs, CSs, and ratios from 2011-2018 in Appendix B.2.6). Then, we used the data of the ratios from 2019-2021 and did the following calculations. First, due to lack of data on charging stations globally, we assumed the US ratio is equal to the global ratio of EVs to CS. Then, we divided the global ratio of EV to CS by the number of EVs globally (collected from previous question) to get the number of charging stations globally. Then, we calculated the change in number of CSs across 2 years. This change demonstrates the number of CSs we need to add per year, which helps us identify the marginal cost per year of CSs globally:

Table 3: Ratio of EV:CS, Number of CS, Change in Number of CS from 2019-2021

Year	Ratio of EV:CS	Number of CSs	Change in Number of CSs
2019	17.58	394374	+ 110040
2020	19.82	527664	+ 133290
2021	22.25	691258	+ 163594

With this data of number of new CS per year, we continued to calculate the total cost (TC)'s two components total fixed costs (TFC) and total variable costs (TVC) (Figure 8). Here, the TFC refers to the amount required to set up one charging station, and the TVC refers to the electricity cost per unit electricity consumed. To calculate the TFC, we divided the number of charging stations into public charging stations (85%) and private workplace charging stations (15%). We further divided these into different levels (L1-L3 in increasing speed of charging) [29]. We determined a unit price for each of the levels shown in the following table [31] [32]:

Table 4: Unit Price and Costs for L1-L3 CS

Public Level	Unit Price ($)	Additional Costs	Private Level	Unit Price ($)	Additional Costs
Public L1	$1,000	Level 1 from $0 to $1000 Public requires pedestal & POS[13] About $100 maintenance fee	Private L1	$500	Level 1 from $0 to $1000 Private requires data-recording About $100 maintenance fee
Public L2	$3,300	Level 2 from $500 to $6000 Public requires pedestal & POS	Private L2	$2000	Level 2 from $500 to $6000 Private requires data-recording
Public L3	$30,000	Level 3 from $10,000 to $40000 Public requires pedestal & POS Significant maintenance fee	Private L3		Unused

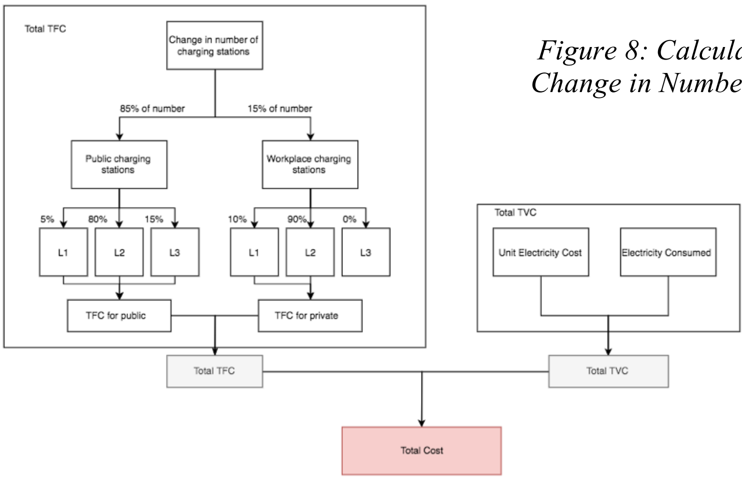


Figure 8: Calculating Total Cost From Change in Number of Charging Stations

Multiplying this unit price by the number of charging stations of each kind, we got the TFC for private and public and combined them to get the total TFC. For TVC, we got the unit electricity cost per kWh (assumed to be constant $0.11/kWh [30]), and the electricity consumed globally per year (from Q1) and multiplied them together to get the total TVC. Finally, we added TFC and TVC to get TC:

Table 5: Total Number of CS in L1-L3 and Costs

Year	Total No. of CSs	No. of Public CS			No. of Private CS		TFC ($)	TVC ($)	TC ($)
		CS L1	CS L2	CS L3	CS L1	CS L2			
2019	110040	4677	74827	14030	1561	14855	702996600	138579205	841575805
2020	133290	5665	90637	16994	1999	17994	851574600	209041107	1060615707
2021	163594	6953	111244	20858	2454	22085	1,045,195,200	307444315	1352639515

3.1.2 Space Costs—Space hogging refers to one single person taking up a large amount of space in a public property. For most public places, this is not a big problem, as places such as malls or airports are large enough that no single person can appreciably affect the area available for other guests [51]. Also, for EVs, there are laws[14] known as "anti ICE-ing Laws" against staying at a charging station once one finishes using it [52]. Hence, space costs are negligible to our cost model.

3.1.3 Security Cost—The largest security consideration is car theft. When cars are left to charge, they are sometimes left unattended. Hence, there is an inherent risk carried in the action. However, this risk is often more on the burden of the owner of the vehicle, as most public charging ports have clauses about personal responsibility of vehicles [54]. Hence, the cost is not significant to the overall cost of the company in charge of the station [44]. In addition, theft of electric vehicles is not a large issue, as a majority of the 10 vehicles with the lowest whole-vehicle theft claims are EVs [53].

[13] POS: point of sale terminal; an electronic device used to process card payments at retail locations
[14] US States that have enacted these laws include California, Oregon, Washington, Florida, and 7 more

3.1.4 Convenience Benefit (negative costs)

The main formula that we used to calculate the convenience benefit is:

$$TB_{Conv} = \bar{s}\times P_0\times EV\%(Y,E,A)\times(1-e^{-\frac{n_{Plug}}{\tau}})$$

Where $\bar{s}$ is the average spending of a consumer, P_0 refers to the maximum population, or peak population, of the public place; $EV\%(Y,E,A)$ is the probability of a randomly selected person owning an EV, and τ is the constant such that τ EV charging stations would attract 63%, or $100(1-e^{-1}\%)$ of peak population. To compute the convenience cost of EVs, we used the following formula:

$$\%EV(Y,E,A) = \beta\times\%EV(Y)^{e_Y}\times\%EV(A)^{e_A}\times\%EV(E)e_E$$

Which is derived from the Codd-Douglas production function [67]. In the formula above, our objective function was the likelihood of a randomly selected person having an EV, which was dependent on the income (Y), age (A), and education level (E) of the person. The results were then raised to the power of the elasticity of input, respectively. In specific, the elasticity was calculated by such [67]:

$$e_x|_{x=0} = \frac{d\%EV(x)}{dx}|_{x=0}\times\frac{x}{\%EV(x_0)}$$

The coefficient β is defined such that the combination of inputs (Y_1, E_1, A_1) that would each lead to a desirable output at the regional average ($\%EV(Y) = \frac{EV(n)}{P(n)}$, where $EV(n)$ and $P(n)$ are number of EVs and population of a region in a certain year) would also result in the regional average, i.e. $\%EV(Y_1,E_1,A_1) = \%EV(Y_1) = \%EV(E_1) = \%EV(A_1) = \frac{EV(n)}{P(n)}$. After collecting relevant data about the demographics and EV statistics [68], we created two regression graphs: relationship between the input and the percentage of EV owners which exhibits the characteristics, or:

$$\%\ of\ EVs = f(Y) = f(E) = f(A)$$

And the relationship between the demographic data depending on the inputs:

$$\%\ of\ population = g(Y) = g(E) = g(A)$$

In order to find the percentage of EVs as a function of each input, we did the following calculation:

$$\%EV(input) = \frac{EV(input)}{Population(input)} = \frac{f(input)\times EV(n)}{g(input)\times P(n)}$$

- Education: $\%EV(E) = \frac{1}{0.7417E^{-2.289}\times 0.756\times\sqrt{2\pi}}\times e^{-\frac{(E-3.113)^2}{2\times 0.756^2}}$
- Age: : $\%EV(A) = \frac{\%EV(A)=\frac{100}{5.54\times\sqrt{2\pi}}\times e^{-\frac{(A-31.167)^2}{2\times 5.54^2}}}{0.7806\times e^{-\left(\frac{y+2.061}{42.14}\right)^2}+1.442\times e^{-\left(\frac{y-40.69}{29.5}\right)^2}}$
- Income: $\%EV(Y) = \frac{\frac{1}{0.545Y\times\sqrt{2\pi}}\times e^{-\frac{(\ln Y-3.35429)^2}{2\times 0.545^2}}}{\frac{1}{1.015Y\times\sqrt{2\pi}}\times e^{-\frac{(\ln Y-2.569)^2}{2\times 1.015^2}}}$

After finding out the percentage of EVs as a function of input, we found the first derivative of the input function to get the functions for elasticity of input.

Then, we inputted the function of $\%EV(input)$ to figure out the input value for a result of $\frac{EV(n)}{P(n)}$. After obtaining the respective inputs (Y_1, E_1, A_1) and output $\%EV(input_1)$, we calculated the point elasticity for each input at the point $(input_1, \%EV(input_1))$. Below are the calculation results:

$$e_Y = -5.564540\times10^{-7} \quad e_E = 3.636997\times10^{-3} \quad e_A = -1.460129\times10^{-5}$$

$$\beta = 0.00218643^{0.996378} = 2.23548\times10^{-3}$$

Once the percentage of EV ownership based on the consumer profile is determined, we had to use that number to calculate the marginal result of adding one additional charging station. To do so, we considered a variety of functions. We knew that the function should increase rapidly initially, before gradually slowing to approximate an extreme value. In the end, we took inspiration from the title of the problem and applied the equation of a charging capacitor:

$$Q = Q_0(1 - e^{-\frac{t}{\tau}})$$

We converted this equation to better fit our needs by changing some of the variables, with P_{att} referring to the attracted number of people, P_p referring to the total population, $\%EV$ referring to the percentage of people with EVs in the area, CS referring to the current number of charging stations, and α referring to the coefficient to reflect the responsiveness of the change.

$$P_{att} = P_p\times\%EV\times(1 - e^{-\frac{nCS}{\alpha}})$$

To calculate marginal results, we simply incremented CS by 1:

$$P_{att} = P_p\times\%EV\times(1 - e^{-\frac{CS+1}{\alpha}})$$

We knew that a feasible number for total current charging stations at a decently sized mall was 5, and from this number we calculated α to equal 5, as this would indicate 63% of EV drivers currently shopped at the mall with the free charging, as well as increasing the number of charging stations would equal 6.67% more of these drivers deciding to switch malls. Both of these numbers align with real life estimates. The formula $R_{inc} = P_{att}\times\bar{s}$ shows that, multiplying this value by the average spending of consumers $\bar{s}$, we calculate the total increase in revenue R_{inc}. In this way, if we have the number of charging stations and population of an area, combined with the consumer profile of income, age, education, and average spending, we can accurately determine the increase in spending from the marginal adding of one charging station.

3.1.5 How it is paid?

Monetary Costs—The bearer of monetary cost depends on the type of costs. Since TC=TFC+TVC, we can separate the cost into two parts. Total fixed costs (costs of installation) is paid solely by the businesses. No matter if the area is a workplace or a public charging station, rational managers of businesses compare the marginal cost of the cost of installation and the cost of electricity with the marginal benefit of the implicit benefits brought by the addition of an electrical outlet [55]. However, the variable cost is sometimes paid by the consumers through other costs (e.g.: additional parking permit payments at a public parking lot [56]).

Space Costs—EVs occupying charging stations will have a greater impact on producers than on consumers. For consumers, there are alternative charging stations [57]; however, for public places, especially for free public charging places one EV hogging the spot for a long time would result in a business losing the potential revenue. However, this cost is minimal as previously mentioned [52].

Security Cost—For EVs, the burden on security cost lies on both sides. On the consumer side, car thefts (though unlikely) would result in heavy monetary cost as well as the time cost and inconvenience of buying a new car [58]. For firms, car theft would result in damaging the reputation of public places, adversely reducing the potential traffic and profitability of the business [59].

Convenience Benefits—Those on the receiving end of these benefits are mainly producers of nearby stores that gain from the presence of these CSs that encourage more consumers to spend at nearby places while they wait for their cars to charge [57].

3.2 Costs for Devices How They Are Paid

3.2.1 Monetary Costs

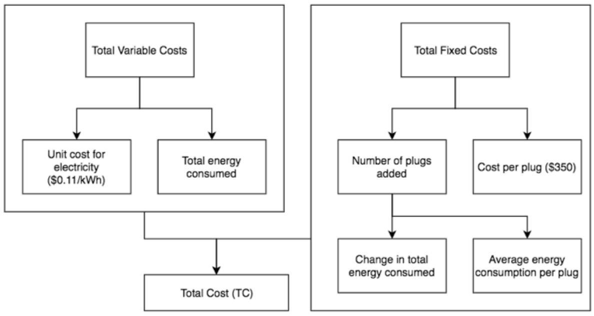


Figure 9: Total monetary cost breakdown for mobile devices

The figure above shows our total monetary cost breakdown for mobile devices. Like with EVs, we split the costs into variable costs and fixed costs. In a simpler manner, we used the equation:

$$TC = TFC + TVC = C_{Plug} \times n_{Plug} + C_{Elec} \times E_{Dev}$$

In the equation above, C_{Plug} and C_{Elec} refer to the unit cost of electrical plugs and electricity respectively, n_{Plug} represents the number of plugs added every year, and E_{Dev} refers to the total energy consumption by charging. Breaking down the equation, we need to calculate TFC and TVC respectively. For TVC, we multiplied the unit cost for electricity (assumed to be the same $0.11/kWh [30]) by the total energy consumed by charging mobile devices in public places (from Q1 data). Second, to calculate TFC, we first found the unit cost per plug of $350 [35]. Then, we calculated the number of plugs added every year by the equation on the left. Then, we used the data on total energy consumption from Q1 and to get the formula:

$$n_{Plug} = \frac{\Delta E_{Dev}}{\bar{E}} \qquad \Delta E_{Dev} = E_{Dev}(n) - E_{Dev}(n-1)$$

By doing so, we found the difference between 2018-2019, 2019-2020, and 2020-2021 to get the change in energy consumption. To obtain $\bar{E}$, we multiplied the number of hours by E_{Dev_U}, the average hourly energy consumption of a plug. Using the formula: $\bar{E} = h \times E_{Dev_U}$. Assuming the outlet is used 7 hours/day, a total of 2555 hours are used per year, or 2562 hours in year 2020. As phones consume 5.5W per hour and laptops consume 75W per hour [20][21], we calculated the weighted average to get an average of $E_U = 20W$. After that, we divided it by the average energy consumption ($\bar{E}$) per plug. The numerical process and final results of TVC, TFC, and TC are shown in the table:

Table 6: Data Process: Finding Costs for Mobile Devices 2019-2021

Year	C_{Plug}	n_{Plug}	C_{Elec}	E_{Dev}	TVC	TFC	TC
2019	$350	1160991	$0.11/kWh	4339761324	477373745.6	406346850	883720595.6
2020	$350	5514343	$0.11/kWh	4621544275	508369870.3	1930020050	2438389920
2021	$350	10265865	$0.11/kWh	5146130000	566074300	3593052750	4159127050

3.2.2 Space Costs—Though space costs are negligible for EVs, for smaller-sized establishments like coffee shops that are heavy in mobile device users, these space-hoggers can greatly reduce the total potential revenue, as other potential customers will not be able to charge their devices [50]. In fact, these free charging ports are one of the most attractive features of a shop to space-hoggers [36]. As a result, as more charging ports are added in response to the increasing demand for energy, the implicit costs a shop will experience will also increase. We calculated this cost using the process as follows:

C_{seat} equals the cost per seat in the shop per hour, and t equals the time hogged in hours. To calculate C_{seat}, we analyzed the data available to us [45]. $T_{revenue}$ represents the average total revenue earned by this type of shop, and T_{seats} represents the total number of seats in the store.

$$T_{cost} = C_{seat} \times t \qquad C_{seat} = \frac{T_{revenue}}{T_{seats}} \qquad T_{cost} = \frac{T_{revenue}}{T_{seats}} \times t$$

Next, we used a probability function to get the number of people hogging a shop at any given time:

$$P(Hog) = P(H_1, H_2, T_{cost}) = H_1 \times H_2 \times T_{seats}$$

This function took the form of the product of the probability that a store would have a people hogging it (H_1), the percentage of seats that are usually hogged by these people (H_2), and the total number of seats (T_{seats}). Combining all the functions:

$$T_{cost} = \frac{T_{revenue}}{T_{seats}} \times t \times P(Hog)$$

$$\therefore T_{cost} = \frac{T_{revenue}}{T_{seats}} \times t \times H_1 \times H_2 \times T_{seats} \qquad \therefore T_{cost} = T_{revenue} \times t \times H_1 \times H_2$$

To test our model, we considered the average coffee shop in a larger city, so as to fit with our data type of choice. The average total revenue of such a shop would be approximately \$260,000 per year, and there would be around 25 seats in such a small-scale store [37] [38].

$$C_{seat} = \frac{T_{revenue}}{T_{seats}} = \frac{260000}{25} = \$10{,}400 \; per\ year$$

$$\$10{,}400 \; per\ year \div 365 \frac{days}{year} \div 10 \frac{hrs}{day} = \$2.85 \; per\ hour$$

Considering that 12.5% of seats are occupied for the full 10 hours per working day for an average coffee shop, we substituted that data into the equation [39].

$$T_{cost} = C_{seat} \times H_2 \times T_{seat} \times t \qquad \therefore T_{cost} = 2.85 \times 12.5\% \times 25 \times 10 = \$89/day$$

Factoring in that only 80% of coffee shops experience significant space hogging and multiplying to compare with total year revenue [40]:

$$T_{cost} = T_{cost} \times H_1 \qquad \therefore T_{cost} = 89 \times 0.8 = \$71.2/day = \$25988/year \qquad \frac{25988}{260000} = 9.95\%$$

From our calculations, we determined that the average coffee shop will lose 9.95% of its total revenue to space cost losses. Seeing as the actual estimated losses to space-hoggers is 10%, our calculated model is acceptably close, being within a 0.5% percent loss [41].

3.2.3 Security Cost—With the increasingly rapid increase in demand for energy and subsequent increase in public free charging, there has also been a related increase and development of new risks associated with these new trends. One such risk is the possibility of juice jacking, a method in which a hacker will take advantage of the fact that both data and power travel along the same cord, meaning that a hacker can potentially transfer data to and from a device while said device is being charged [42]. We calculated the potential cost of this juice jacking using the formula below:

$T_{cost} = \alpha \times P(JJ, C) \times T_{phones}$. In this formula, the total cost equals the probability of a charging station being used maliciously (JJ) combined with the probability that someone would use that specific charging station without questioning its security (C). This probability would then be multiplied by a coefficient α to reflect the growing ability of phones to negate such an attack. To model this data, we used the only known large scale juice jacking test, performed by Wall of Sheep. They set up a free charging station that they had infected with juice jacking software at a conference of, at the time the experiment was conducted, 10,000 people [43]. Of these 10,000 people, approximately 360 of them directly used the charging station without even asking about the security risks, yielding a probability $P(JJ, C)$ of:

$$P(JJ, C) = \frac{360}{10000} = 3.6\%$$

Assuming a relatively uniform distribution of people, this $P(JJ, C)$ can be applied to the larger population. After calculating this probability value, we had to determine the coefficient of this probability that would be applied to account for the increasing defensive prowess of phones. We set this coefficient to equal 1%, for there is no real data on this, but we do know that the value should approach zero; most modern phone models can only pass data along the charging line after the user explicitly changes a setting. Therefore, $T_{cost} = 0$. There is the second security concern of devices being stolen while unattended. However, like the EVs, this is the sole responsibility of the owner, and hence, does not contribute to a significant amount of the cost of a company/shop that provides the charger.

3.2.4 Convenience Cost (Benefit)—For mobile devices, it is difficult to quantify the marginal benefit of one additional outlet. Hence, we can only figure out the general benefit of outlets as a whole to a retail store. Of course, we are aware that the addition of numerous outlets does include multiple advantages, as studies have shown that customers who charge their phones stay in the store 2.27 times longer than those who don't; there's a 54% increase in conversion for customers who charge their phones; and customers who charge their phones spend 29% more than those who don't [50]. However, we do not know the specific quantity that must be reached (i.e.: the threshold quantity) in order for devices to bring such advantages to stores. One additional outlet likely does not do a lot, but, say, 50 more outlets would likely bring many benefits. To solve this issue, we used the previous convenience benefits for EVs. Though EVs and devices themselves are fundamentally different, the concept of convenience benefits for both are the same. Hence, we decided to multiply a coefficient in front of the convenience benefits for EVs to get the cost for devices. This coefficient is 0.01, because the total amount of charging sockets at Heathrow is more than 3000. However, an airport has an average of 30 EV charging stations [63]. Hence, the estimated ratio is 0.01.

3.2.5 How It Is Paid

Monetary Costs— Monetary costs for devices, like the EVs, can also be separated into fixed costs and variable costs. However, unlike the EVs, the businesses will almost always bear the whole burden of monetary cost [60]. They, not the consumers, pay to hire the electrician and buy the materials for installation of electrical outlets. They also must pay for the variable costs of electricity, as the energy consumed is recorded in the businesses' electricity bill [60].

Space Costs—Businesses also take the majority of the cost of space hogging. However, this cost does not show up as a negative on the accounting book, with it being paid in the form of decreased venue on behalf of the shop [60]. As more seats are hogged up, the total revenue of the shop decreases; there is no net loss in revenue because of seat hogging.

Security Cost— In contrast, the security cost of these public charging stations is almost entirely on the users, for it is the responsibility of these people to take care of their own belongings [61]. For most businesses, they do not claim any liability for any stolen or misplaced items, with the total cost of businesses being only the discomfort of hearing consumers complain about the missing items or loss of brand loyalty [60]. Moreover, the burden on consumers may not be in the form of monetary payment, with security issues such as juice jacking potentially leading to greater losses in the form of missing data [62]. In this way, consumers pay for the security issues through their many inconveniences.

Convenience Benefits—The stakeholders that benefit from convenience benefits are the producers, as they reap the positive externalities of these outlets. As previously mentioned, the presence of outlets would increase sales, build engagement and customer loyalty with better onsite service, market to consumers in and out of the store, and help stores gain greater appeal [50].

4. Question 3: Changes in the Cost Model

4.1 Changes in EVCS Costs

4.1.1 Monetary Costs—To determine the change in the monetary cost in different public places, we can consider TFC and TVC separately. TFC is dependent on the unit cost of an EV outlet and number of EV outlet. This, in turn, depends on the demand for charging powers of EV outlets and total demand for EV charging, respectively. If the consumers demand high charging power, then the businesses (e.g.: airports, train stations) have to supply more level 2 or even level 3 EV outlets, while public places with lower power demand (e.g.: coffee shops) can lower the costs by using more level 1 AC charging stations [56]. Similarly, public places with higher electricity demand usually face both a higher fixed cost and a higher variable cost than public places with lower demand for electricity due to increased demand for more EV outlets and increased electricity consumption [57].

4.1.2 Space costs–Although space costs for EVCS are less significant, the situation still differs between public places for electric vehicles. In this situation, the cost of space hogging is dependent solely on the number of charging stations [52]. The cost of space hogging in public areas with less charging stations will be a lot greater of that of the public areas with more charging stations.

4.1.3 Security Costs—Different public places have different security costs for electric vehicles, although the difference is sometimes negligible and hard to calculate. The main factor of the security cost will be the level of security of the public places. If the place itself has higher levels of security (usually a government owned institution such as an airport), then it likely contains a more holistic security protocol (i.e.: security cameras, guards, etc.) [63]. However, if the place has generally has a lower level of security, they might not be willing to spend more budget on security and hence will have a higher risk.

4.1.4 Convenience Benefits— As the function is given by $\Delta R = \bar{s} \times P_0 \left(1 - e^{-\frac{n_{plugs}}{\beta}}\right)$ there are a lot of factors of influence. Public places with both a higher general average spending, population base, and number of EV outlets will result in a higher convenience benefit for businesses. Hence, locations with a greater population density (i.e.: metropolitan areas), will have a greater benefit than locations with lower population density (i.e.: rural areas). Similarly, EVCSs in wealthier neighborhoods will have a greater marginal benefit from convenience because its population will be more willing to spend.

4.2 Changes in Devices Station Costs

4.2.1 Monetary Costs—Again, we separate the total cost into fixed cost and marginal cost. Fixed cost is directly proportional to the number of plugs installed, which is a function of energy consumed; meanwhile, variable cost is directly proportional to the electricity consumed, assuming the unit cost holds constant. Hence, demand for electricity for device charging is the sole determinant of a company's monetary costs for devices.

4.2.2 Space Costs—Similar to the space costs for EV, the cost of space hogging for electronic devices is dependent on the number of charging outlets for devices [60]. The cost of space hogging in public areas with less charging stations bear higher costs for space hogging than the ones with more outlets.

4.2.3 Security Costs—Different public places have different security costs for electric vehicles, although the difference is sometimes negligible and hard to calculate [64]. Similar to security costs for electric vehicles, leaving a phone or a laptop at a public charging place will result in serious consequences, especially because stealing a personal device is easier than stealing an EV [65]. Traffic also plays a role in determining the security costs, as public places with more traffic tend to have a higher chance of theft [66].

4.2.4 Convenience Benefit—This is similar to EVs, because as general population density and wealth increase, the benefits for producers increase as well. (See 4.1.4 for more details).

5. Question 4: Optimizing the Cost Model

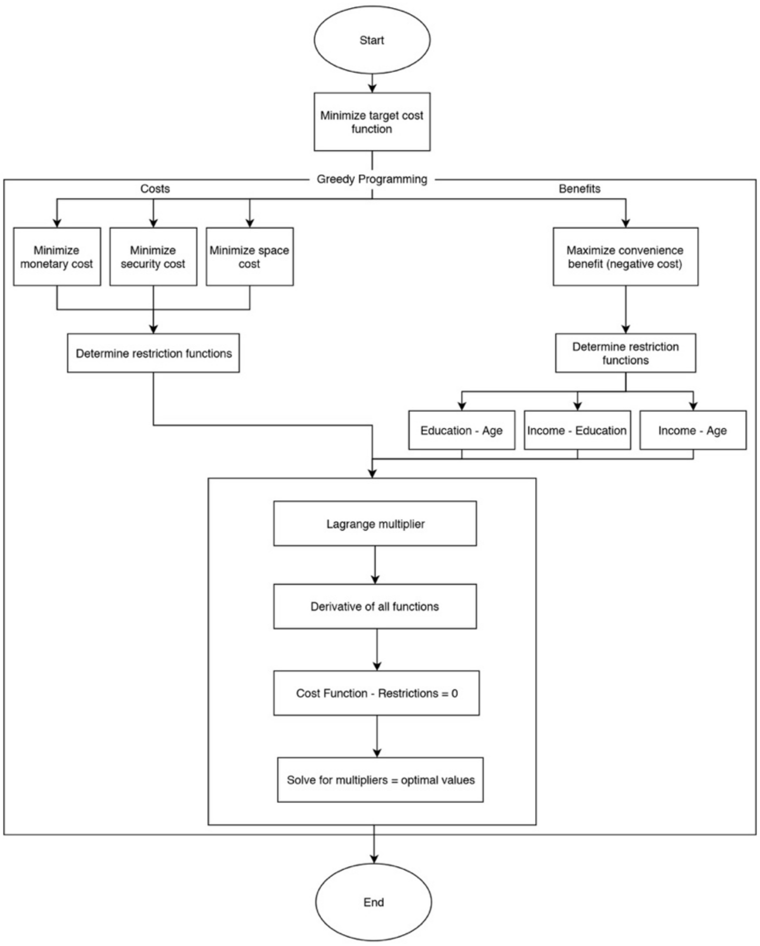


Figure 10: Breakdown of Optimization of Costs with Lagrange

For Q4, we recognized that an optimization of the cost function was required. As such, our first step was to split the net cost function back into its components [69].

$$T_{c1} = Conv, T_{c2} = MC, T_{c3} = SH, T_{c4} = SC$$

Then, we implemented the principle of Greedy programming [70], which states that in many cases, the best solution to a multistep problem can be found by following the best solution for each of the individual steps. In this case, the multistep problem is optimizing the net cost, with the individual pieces of the cost being the individual steps.

5.1 Optimizing Convenience

Out of all of the variables, the convenience cost is unique in that it demands a maximization instead of a minimization to find the most beneficial point. Convenience uses the formula below [69]:

$$P_{att} = P_0 \times \%EV \times (1 - e^{-\frac{CS}{\alpha}}) \times \bar{s}$$

To optimize, we first determined that P_0, $1 - e^{-\frac{CS}{\alpha}}$, and $\bar{s}$ are all not the inputs that would need to be examined in this case, for they are properties of the area itself and cannot be changed in any significant way by policy. Therefore, we focused on the $\%EV$ aspect of the optimization, noting that, because of the way the formula is organized, maximizing $\%EV$ is equivalent to maximizing the whole benefit equation. However, $\%EV$ is split into 3 different inputs itself (Y for income, A for age, and E for education level), leading us to use the method of Lagrange optimization.

$$f(Y, A, E): \%EV = \rho \times Y^{e_Y} \times A^{e_A} \times E^{e_E}$$

From this initial equation, we determined our restrictions through regression, picking quadratic values for their high R-squared values without over-fitting, on other sets of data [69]:

$$g(E, A): E = -0.006367A^2 + 0.4462\text{A} + 29.82$$

$$h(Y, E): Y = 19.01 \times e^{0.2569E}$$

$$j(Y, A): Y = -0.019A^2 + 1.9341A - 11.5204$$

These equations can then be written into a Lagrange function, where the lambda values are Lagrangian multipliers [69]:

$$\mathcal{L}(Y, A, E) = f(Y, A, E) - \lambda_1 \times g(E, A) - \lambda_2 \times h(Y, E) - \lambda_3 \times j(Y, A)$$

$$\nabla g(E, A), \nabla \text{h(E, A)}, \nabla \text{j(Y, A)} \neq 0$$

To optimize the equation, we need to determine where the gradients are parallel by finding the partial derivatives of all of the equations:

$$\nabla_{Y,A,E} f(Y, A, E) = (\frac{\partial f}{\partial Y}, \frac{\partial f}{\partial A}, \frac{\partial f}{\partial E}), \qquad \nabla_{Y,A,E} g(Y, A, E) = (0, \frac{\partial g}{\partial A}, \frac{\partial g}{\partial E}),$$

$$\nabla_{Y,A,E} h(Y, A, E) = \left(\frac{\partial h}{\partial Y}, 0, \frac{\partial h}{\partial E}\right), \qquad \nabla_{Y,A,E} j(Y, A, E) = (\frac{\partial j}{\partial Y}, 0, \frac{\partial j}{\partial A})$$

Substituting values into the equations and rewriting the format

$$\nabla_{Y,A,E} f = \begin{bmatrix} \rho e_Y Y^{e_Y - 1} A^{e_A} E^{e_E} \\ \rho e_A A^{e_A - 1} Y^{e_Y} E^{e_E} \\ \rho e_E E^{e_E - 1} Y^{e_Y} A^{e_A} \end{bmatrix} \qquad \nabla_{Y,A,E} g = \begin{bmatrix} 0 \\ 0.1273A - 0.4462 \\ 1 \end{bmatrix},$$

$$\nabla_{Y,A,E} h = \begin{bmatrix} 1 \\ 0 \\ -0.488e^x \end{bmatrix} \qquad \nabla_{Y,A,E} J = \begin{bmatrix} 1 \\ 0.0388A - 1.9341 \\ 0 \end{bmatrix}$$

Therefore, the tangency condition ends up looking like:

$$\begin{bmatrix} AE \\ IE \\ IA \end{bmatrix} = \lambda_1 \begin{bmatrix} 0 \\ 0.1273A - 0.4462 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 \\ 0 \\ -0.488e^x \end{bmatrix} + \lambda_3 \begin{bmatrix} 1 \\ 0.0388A - 1.9341 \\ 0 \end{bmatrix}$$

Solving this equation with the other restriction equations yields the optimal values in regards to the specific inputs. In regards to how exactly this model can be optimized in real life, businesses will now have a target audience for their products, seeing as it is these calculated people who are most likely to enjoy the company's products. Therefore, increased attraction directed at this group of people can actually lead to great returns in the larger picture of company revenue, for they will be able to offset more of the other three costs.

5.2 Optimizing Monetary Cost

We applied the same general Lagrangian method to the monetary costs, with only the functions and restrictions changed to reflect the difference in aspect.

$$f(E_V, n_p): T_c = n_p \times c_p + E_{EV} \times c_E$$

In this case, we explore the values of n_p and E_{EV} in details, for c_p and c_E ae just coefficients that reflect prices. For our restrictions, we considered that a change in energy consumption E_{EV} would lead to a relatively large change in n_p to meet this demand, while the inverse is not true because the amount of energy consumed does not depend on the number of available ports. However, both variables must rely on another input of population (p), that is determined in a specific area's information. As such, our restriction function and derived Lagrange formula as follows:

$$g(E_V, n_p): E_{EV} = (n_p)^2 \times p \qquad \mathcal{L}(E_V, n_p) = f(E_V, n_p) + \lambda \times g(E_V, n_p)$$

Applying Lagrange multipliers and solving for the partial derivatives,

$$\nabla_{E_V, n_p} f(E_V, n_p) = \left(\frac{\partial f}{\partial E_V}, \frac{\partial f}{\partial n_p}\right) = [{}^{c_p}_{c_E}]$$

$$\nabla_{E_V, n_p} g(E_V, n_p) = \left(\frac{\partial g}{\partial E_V}, \frac{\partial g}{\partial n_p}\right) = [{}^{2n_p}_{1}]$$

Finally, the final value calculated by solving all of these equations is:

$$\begin{cases} n_p = \dfrac{c_p}{2c_E \times p} \\ E_{EV} = \dfrac{(c_p)^2}{4(C_e)^2 \times p} \end{cases}$$

The optimization of this problem is also somewhat similar, with us having control over how many sockets we can buy and how much energy we can consume. Approaching these values will, in reality, greatly contribute to the reduction of monetary costs, with these ideal values resulting in the lowest possible cost of energy, which is obviously good for businesses.

5.3 Optimizing Space Costs

In much the same way as the previous two, optimizing space costs also consists of a target cost function, requirement, and a Lagrangian solution: $f(x): T_c = H_1 \times H_2 \times T_R \times t$.

For this specific situation, we focused on the variables H_2 (the probability a specific chair will be hogged) and t (time) as the other variables are properties of the area itself. For the restriction of this function, we considered that the probability H_2 would increase at a much faster rate than the time, for if people spend more time at a specific place charging, then intuitively, there will be a build up of people, causing the probability to drastically increase. This relationship is represented by the equation below where c is a constant: $\frac{t}{e^{H_2}} = c$.

When optimizing this equation, it can be seen that it does not really bound the function, meaning that the smallest possible values of f(x) will exist at the natural extrema. Indeed, in this case, the most cost-optimizing choice is to try to reduce H_2 to zero, which aligns with the already understood correlation between a decrease in revenue and the total number of seat hoggers. We can directly apply policies that ban people from staying in shops after a certain amount of time, and with firm enough enforcement of these policies, we can minimize the amount of time and number of seats that these people take up. As such, the cost function is more gradual, for the space cost is dramatically reduced.

5.4 Optimizing Security Costs

In contrast to the other two equations, we did not apply any restriction equations or variable in solving the answer. Indeed, the only way to reduce this cost is to tell people to be more attentive and careful of their own belongings. This would most likely lead to a rise in public safety for personal items, which is the largest reason for the security cost of electronic devices and sockets. Other possible initiatives are similar to the ones that companies have already enacted in removing the possibility for data to pass along the charging line. In this way, dangers such as juice jacking can be completely eradicated.

6. Sensitivity Analysis

We conducted a sensitivity analysis on the Monetary Cost Model, testing the total cost:

$$TC = TFC + TVC = C_{Plug} \times n_{Plug} + C_{Elec} \times E_{EV}$$

Here, we can only change the number of EV charging stations, or n_{Plug}. If we adjust the change in number of CSs by 5%, we get the following, which we plugged into our model to derive Table 7:

Table 7: Monetary Cost Sensitivity

Year	Change in number of CS +5%	Change in number of CS – 5%
2019	+115542	+104538
2020	+139955	+126626
2021	+171774	+155414

Table 8: Results of sensitivity test for monetary cost model

Year	New TFC	New TC	Change	New TFC	New TC	Change
2019	738197838	876777043	+4.18%	667893282	806472487	-4.17%
2020	894172495	1103213602	+4.02%	809013514	1018054621	-4.01%
2021	1097464086	1404908401	+3.86%	992940046	1300384361	-3.86%

The results of the analysis indicate that the model of total monetary costs is not very sensitive to changes in number of charging stations, with the change in result being less than the change in input. This aligns with our expectations, because the increase in number of charging stations demanded is not enough to appreciatively affect the market for these products; the output changes little.

Table 9: Results of sensitivity testing for space cost model

Variable	+5%	Percent Change	-5%	Percent Change	Original Result
H_2	\$93.51/day	5.07%	\$84.6/day	-4.93%	\$89/day
Time	\$93.52/day	5.07%	\$84.61/day	-4.93%	\$89/day

In regards to the space cost model, we first began with the sample calculation we provided in the paper, before exploring the variables we had explicitly pointed to. As can be seen, both of the variables have the same effect on the end result, which makes sense considering that they talk about fundamentally the same phenomena but from two different angles, with one being probability and another an average. In addition, both of these factors impact the model by more than the amount that they are changed, indicating that our model is relatively sensitive to changes these variables, which is intuitive based on how the problem of space hogging is defined by how much time is spent.

Table 10: Result of sensitivity testing for convenience model

Variable	Percent Change	Percent Change	Original Result
Y	7.6%	-7.6%	1.3%
A	3.7%	-4.1%	1.3%
E	5.05%	-5.61%	1.%
β	-2.54%	3.33%	63%

In regards to our convenience model, we first analyzed the results in regards to the three main inputs that make up our consumer profile. For the income, we determined that this would actually greatly affect the determined percentage, in accordance with our research on consumer preferences. On the other hand, the results for age and education are much smaller, with especially age being not as important of a factor; more distinguished elder citizens also seem to enjoy using EVs and new devces. After that, we also took an in-depth view of the variable beta, and how changing that coefficient would affect the result. As can be seen, changing that coefficient does not affect the end result more than the variable itself was changed, meaning that the model is relatively insensitive towards this data point. This implies that small fluctuations in timing will not deter the trend of electric vehicles and the benefits of driving these cars.

7. Model Testing

To test out our model on the consumer profile of convenience benefits, we chose a random sample of a combination of income, education level and age: $(Y, E, A) = (17,2,22)$. As all three values are similar to the value of 1.2% from the regression model, we expected a result that was similar to 1.2. According to our calculation (calculation process will be included in the appendix), we obtained a result of 1.1368%, which was very close to the estimated data of 1.2%. The percent error is calculated below:

$$\%\ Error = \frac{|1.1368 - 1.2|}{1.2} \times 100\% = 5.27\%$$

We thought this was a relatively good result, considering the level of detail described by our model. However, the fact remains that this error could be reduced, if we had considered more variable, such as, but not limited to gender, race, and cultural differences. Despite this, we still believe that our model can significantly represent the larger more important trends in the modern world, for we can predict to a relatively high degree of accuracy the benefits that free energy sources can provide.

8. Conclusion

8.1 Strengths and Limitations

Table 11: Strengths and Weaknesses

Areas	Strengths	Limitations
Research	Our research forms a strong foundation for our paper because it triangulates data from multiple different sources (Appendix A.1). All our conclusions and deductions are based on researched knowledge, so it adds credibility to our later claims. After long hours of searching, we were able to find data on the most significant parts of our model and move on to compile, sort, and analyze them in an effective manner. We also have a total of 70 number of sources, which is a significant amount and reinforces the validity of our model's conclusions.	One limitation is the lack of data on this research topic. It is nearly impossible to find the data on number of charging ports in each small café or airport in China, or energy per mile expended by every EV model from 2011-2018. This had an impact on our model as we had to simplify many areas and reduce certain factors to constants (such as the energy per mile for EVs). This could potentially be solved with access to a larger database, cross-checking data, or averaging the data to get a more accurate model.
Model Design	Our model design is holistic because we considered multiple perspectives and divided energy consumption into many categories and subcategories. We individually considered EVs, mobile devices (phones, and laptops). We took into consideration four different regions (Global, US, China, and Europe) for EVs, and a significant recent year period of 2011-2018 to obtain the most accurate trends. We also predicted three years into the future 2019-2021 and cross-checked it against other predictions to test our model. We also incorporated a complex methodology (i.e.: regression, integration, the LaGrange multiplier, Keynes' Law, etc.) to produce results.	A large part of our data analysis is built on the concept of correlation (e.g.: demand leading to supply and assuming that an increase in EVs will lead to a proportional increase in charging stations). However, we must consider that correlation does not equal causation, and there could be possible third variables that may alter future trends. However, as we are unable to conduct a legitimate experiment that fully controls the environment that the EV charging stations develops in, it is impossible to account for all extraneous variables that may impact our analysis of the data. This limitation applies to a majority of our models, and could be fixed with access to a greater database, so we can add more factors into consideration.
Generalizability	Our model is reasonably realistic because it takes into consideration multiple different regions around the globe instead of just focusing on only the global data. Global data is likely subject to variations due to outliers and does not fully capture the recent patterns of development. Hence, for our EV data analysis, we considered not only the global statistics, but also the statistics of the three largest markets for EVs: China, US, and Europe [5]. Through this, we can cross check the trends for each region and see if they correspond to each other, adding credibility and validity to our model. Hence, with this broader spectrum of considerations, our model is high in generalizability because it is more applicable to the detailed cases.	The model is slightly reductionist because, although we considered multiple regions, they are all developed, wealthy, industrialized countries with access to abundant resources for technological development and security. This is especially significant for the charging stations funded by government institutions (e.g.: those located in airports). Hence, we can only generalize this model to countries in the same category. Given the time to further develop our model, we could also consider countries that are slightly less developed and compare the two to see the difference in costs and optimization. However, this remains an ideal model as there is little data on EVs and devices in less developed countries.

8.2 Reflection

Humans are moving into an increasingly digital age and simultaneously becoming more concerned with energy conservation and preserving a healthy relationship with the Earth. Thus, the topic of free energy consumption is more important than ever. How has this type of energy consumption changed over time? How will it continue to change? What are the costs, benefits, and impacts to various places with these sources? And finally, how do we optimize the costs to ensure maximum benefit to society as a whole?

Our paper offers a realistic, multifaceted model that addresses all these questions. Through regression, integration, and the LaGrange multiplier we have illustrated recent trends of free energy consumption in EVs and mobile devices from 2011-2018, and predicted trends into 2021. We have also found the cost model and analyzed methods to change and optimize it. Although our model faces a fair level of obstacles, specifically lack of available data, our team has found a way to overcome them through justifiable simplification of values into constants and synthesis of research from multiple sources. As a whole, our team has successfully responded to the prompt and created a reliable model that has great applications for the upcoming future.

APPENDIX A

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*All citations throughout the paper are referenced with a bracket [x], where x corresponds to the number in the following list

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A.2 Table of Tables

A.3 Table of Figures

A.4 Variables and Definitions

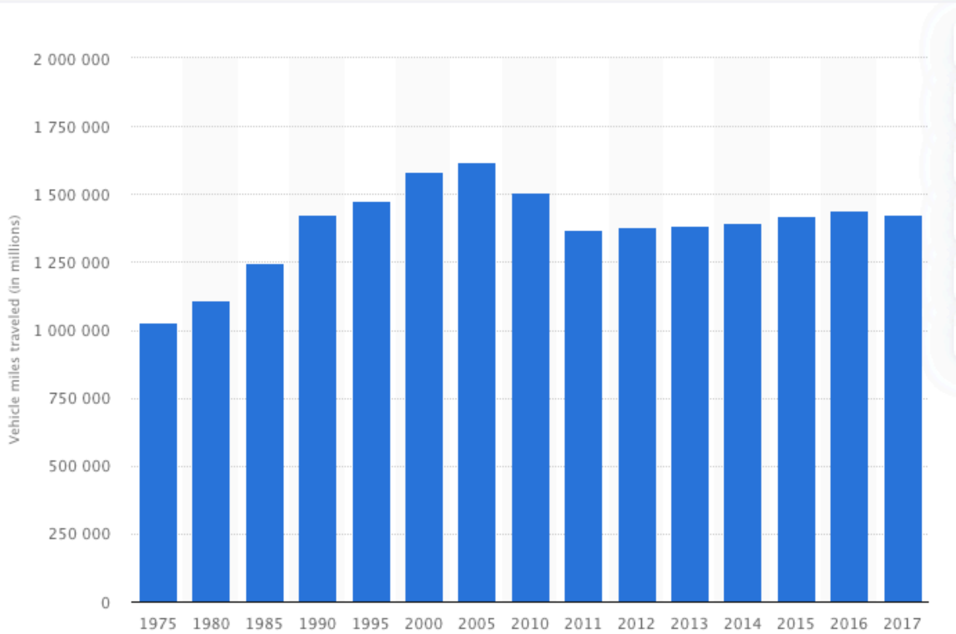
Variables	Definitions
$E(n)$	Total energy consumption of EVs
n_{EV}	Total number of electrical vehicles per year
m_{EV}	Average mileage per year of electrical vehicles
E_{mile}	Average energy per mile
n	Difference between current year and 2010
$EV(n)$	Aggregate number of EVs at n
$E_{PCS}(n)$	Free energy consumption at n
$s(n_{laptops})$	Laptop sales at n
$s(n_{phones})$	Phone sales at n
$E_{total(laptop,phones)}$	Total energy consumption
hr	Average number of hours on average
E_{hr}	Energy use per hour
$n_{Dev(laptop,phones)}$	Number of devices
$E_{Dev(laptop,phones)}$	Energy consumption of devices
E_{Dev_P}	Energy consumption of devices in public
TC	Total cost
TFC	Total fixed cost
TVC	Total variable cost
C_{Plug}	Unit cost per electric plug
n_{Plug}	Number of electric plugs added per year
C_{Elec}	Unit cost of electricity
E_{EV}	Total electrical energy for EVs
E_{Dev}	Total electrical energy for devices
ΔE_{Dev}	Change in electrical energy for devices

$\bar{E}$	Annual average electrical energy per electrical outlet
h	Average charging hours
E_{Dev_U}	Average hourly energy consumption
T_{cost}	Total cost of specific section
C_{seat}	The total cost per seat in a shop
t	Time hogged in hours
$T_{revenue}$	Total revenue made by a shop in a year
T_{seats}	Total number of seats in a shop
$P(Hog)$	The probability that a seat will be hogged at a certain point
H_1	The probability that a store will have seat hoggers at any specific time
H_2	The percent of seats that are occupied at any time
α	The coefficient of phone security
$P(JJ,C)$	The probability that a kiosk will be infected after taking into account the probability of using the infected kiosk
T_{phones}	The total number of phones
P_{att}	The attracted number of people
P_p	Total population
$\%EV$	The percentage of people with EVs in the area
nCS	Current number of charging stations
α	Coefficient to reflect the responsiveness of the change
R_{inc}	The total increase in revenue
$\bar{s}$	The average spending of consumers

Appendix B

B.1 Additional Graphs

Appendix B.1.1 Data On Annual Mileage for the US from 1975-2017



From this graph, we can see that from 2011-2017, there is not a drastic change in annual mileage across time. Hence, we can assume that this value stays constant for our calculations in a given region.

B.2 Additional Data and Statistics

B.2.1 Total Number of EVs from 2011-2021

Year	Global	China	US	Europe
2011	54,300	14,700	24,400	4,000
2012	183,000	32,600	82,200	23,100
2013	366,800	38,600	157,700	67,900
2014	629,900	49,900	245,500	149,300
2015	1,039,200	115,700	349,900	278,300
2016	1,704,900	317,600	485,400	466,100
2017	2,780,100	769,300	676,700	723,800
2018	4,461,200	1,616,700	958,100	1,063,000
2019	6,933,100	3,023,200	1,349,900	1,491,300
2020	10,458,300	5,210,500	1,897,700	2,009,200
2021	15,381,400	8,436,300	2,681,800	2,618,100

B.2.2 Total Energy Consumed 2017-2021 (in kWh)

Year	Global	China	US	Europe
2011	180921840	57403899	95403672	7772978
2012	609493454	127359213	321331219	44781602
2013	1222052477	150909686	616772794	131777880
2014	2098441205	195003494	959939035	289803787
2015	3462006601	452493242	1368261056	540191263
2016	5679600285	1242135964	1898390455	904562214
2017	9261466674	3008506895	2646375427	1404801272
2018	1486178739	6322440007	3746848377	2063144172
2019	2309653415	11822849402	5279063379	2894418536
2020	3484018450	20376738824	7421348674	3899594799
2021	5124071922	32991897464	10487734033	5081390176

B.2.3 Total free energy consumed per region 2011-2021 (in kWh)

Year	Global	China	US	Europe
2011	9046092	2870195	4770184	388649
2012	30474673	6367961	16066561	2239080
2013	61102624	7545484	30838640	6588894
2014	104922060	9750175	47996952	14490189
2015	173100330	22624662	68413053	27009563
2016	283980014	62106798	94919523	45228111
2017	463073334	150425345	132318771	70240064
2018	743089370	316122000	187342419	103157209
2019	1154826710	591142470	263953169	144720927
2020	1742009230	1018836941	371067434	194979740
2021	2562035960	1649594873	524386702	254069509

B.2.4 Total amount of global laptop and cellphone sales (in millions of units) 2011-2018

Year	Global Laptop Sales (millions of units)	Global Operating Laptops (millions of units)
2011	209	752.405
2012	201	793.531
2013	180.9	799.153
2014	174.28	779.348
2015	163.1	744.195
2016	156.8	703.771
2017	161.6	668.153
2018	162.3	647.419

B.2.5 Total Number of EV sales per region from 2011-2018

Year	2011	2012	2013	2014	2015	2016	2017	2018
China [6]	1,000	3,000	3,800	41,000	150,000	282,000	596,000	1,120,000
US [7]	17,000	55,000	96,000	119,000	115,000	157,000	200,000	358,000
Europe [8]	8,900	26,000	62,000	91,000	183,000	224,000	269,000	396,000
Global [9]	54,000	141,000	223,000	341,000	573,000	813,000	1,277,000	2,099,000

B.2.6 Ratio of Charging Stations to EVs in US from 2011-2021

Year	Number of Charging Stations (n_{CS})	Number of EVs (n_{EV})	Ratio of EV:CS
2011	3,394	24,400	7.19
2012	13,392	82,200	6.14
2013	19,410	157,700	8.13
2014	25,602	245,500	9.59
2015	30,945	349,900	11.31
2016	42,029	485,400	11.55
2017	50,627	676,700	13.37
2018	61,067	958,100	15.69
2019	/	/	17.58
2020	/	/	19.82
2021	/	/	22.25

B.2.7 Percent Error of Laptop Sales (Model Predicted-Online Predicted)

Year	Model Predicted Sales	Online Predicted Sales	Percent Error
2019	184.6251	166	10.8%
2020	217.84	181	20.4%
2021	267	203	31.5%

B.2.8 Annual Mileage of Regions (China, US, Europe, Global)

Region	Annual Mileage	Region	Annual Mileage
China [11]	13,500 mi / year	Europe [13]	6700 mi / year
United States [12]	13,500 mi / year	Global [14]	11,500 mi / year

B.2.9 Global Laptop and Phone Sales [and operating] 2019-2021 (millions of units)

Year	Global Laptop Sales (millions of units)	Global Operating Laptops (millions of units)	Global Phone Sales (millions of units)	Global Operating Phones (millions of units)
2019	184.6251	651.646	5.01	1509.3
2020	217.84	690.913	30.14	1520
2021	267	775.297	85.96	1576.5

B.2.10 Ratio of EV:CS, Number of CS, Change in Number of CS from 2019-2021

Year	Ratio of EV:CS	Number of CSs	Change in Number of CSs
2019	17.58	394374	+ 110040
2020	19.82	527664	+ 133290
2021	22.25	691258	+ 163594

B.3 Additional Formulas

B.3.1 Regression formulas for EV Sales in 4 regions (Global, US, China, Europe)

Region	Regression Formula for EV Sales S(n)
China	$S(n) = 5.3658n^3 - 31.6838n^2 + 47.7970n$
US	$S(n) = 1.6715n^3 - 15.3221n^2 + 58.1700n$
Europe	$S(n) = 0.0249n^3 + 5.2340n^2 + 4.5080n$
Global	$S(n) = 7.1795n^3 - 41.8630n^2 + 132.9368n$

B.3.2 Formulas for finding total energy per region

Region	Total Energy E(n) and Factor (mileage per year × energy per mile)
China	$E(n) = 13{,}500 \times 0.289682 \times n$
US	$E(n) = 13{,}500 \times 0.289682 \times n$
Europe	$E(n) = 6{,}700 \times 0.289682 \times n$
Global	$E(n) = 11{,}500 \times 0.289682 \times n$

B.3.3 Regression formulas for laptop and phone sales

$$s(n_{laptops}) = 0.4199n^3 - 4.438n^2 + 3.744n + 204.3$$

$$s(n_{phones}) = 10.3284n^2 - 140.422n + 457.34$$

B.3 Code

B.3.1 Code for Income Simulation

```
clear;
clc;
%Income Function

%DataGeneration
Income = zeros(0);
%1
r = 5*rand(100,1) + 10; %Simulate Income Values with Uniformly Random Number Generator
r = reshape(r,1,length(r));
Income = [Income,r];
%2
r = 5*rand(450,1) + 15;
r = reshape(r,1,length(r));
Income = [Income,r];
%3
r = 5*rand(1400,1) + 20;
r = reshape(r,1,length(r));
Income = [Income,r];
%4
r = 5*rand(1200,1) + 25;
r = reshape(r,1,length(r));
Income = [Income,r];
%5
r = 5*rand(1350,1) + 30;
r = reshape(r,1,length(r));
Income = [Income,r];
%6
r = 5*rand(950,1) + 35;
r = reshape(r,1,length(r));
Income = [Income,r];
%7
r = 5*rand(1000,1) + 40;
r = reshape(r,1,length(r));
Income = [Income,r];
%8
r = 5*rand(800,1) + 45;
r = reshape(r,1,length(r));
Income = [Income,r];
%9
r = 10*rand(900,1) + 50;
r = reshape(r,1,length(r));
Income = [Income,r];
%10
r = 10*rand(350,1) + 60;
r = reshape(r,1,length(r));
Income = [Income,r];
%11
r = 10*rand(300,1) + 70;
r = reshape(r,1,length(r));
Income = [Income,r];
%12
r = 10*rand(350,1) + 80;
r = reshape(r,1,length(r));
Income = [Income,r];
%13
r = 10*rand(300,1) + 90;
r = reshape(r,1,length(r));
Income = [Income,r];
%14
r = 50*rand(250,1) + 100;
r = reshape(r,1,length(r));
Income = [Income,r];
%15
```

```
%6
r = 5*rand(950,1) + 35;
r = reshape(r,1,length(r));
Income = [Income,r];
%7
r = 5*rand(1000,1) + 40;
r = reshape(r,1,length(r));
Income = [Income,r];
%8
r = 5*rand(800,1) + 45;
r = reshape(r,1,length(r));
Income = [Income,r];
%9
r = 10*rand(900,1) + 50;
r = reshape(r,1,length(r));
Income = [Income,r];
%10
r = 10*rand(350,1) + 60;
r = reshape(r,1,length(r));
Income = [Income,r];
%11
r = 10*rand(300,1) + 70;
r = reshape(r,1,length(r));
Income = [Income,r];
%12
r = 10*rand(350,1) + 80;
r = reshape(r,1,length(r));
Income = [Income,r];
%13
r = 10*rand(300,1) + 90;
r = reshape(r,1,length(r));
Income = [Income,r];
%14
r = 50*rand(250,1) + 100;
r = reshape(r,1,length(r));
Income = [Income,r];
%15
r = 50*rand(200,1) + 150;
r = reshape(r,1,length(r));
Income = [Income,r];
%16
r = 50*rand(100,1) + 200;
r = reshape(r,1,length(r));
Income = [Income,r];
Income = Income/2;%Change per family to per person
Income = Income *10000; %Change unit from 10K CNY to 1CNY
Income = Income/7000; %Change Unit from 1 CNY to 1 USD

%Graph Drawing
histogram(Income,50)

%Fitting
histfit(Income,50,'lognormal')

%Finding Graph Parameters
i = reshape(Income,10000,1);
pd = fitdist(i,'Lognormal')

%Function Creation
x = 10:1:180;
out = pdf(pd,x);
plot(x,out)
```

B.3.2 Codd Dogulas Code

CoddDogulas.m | StanleyDataProcess.m | Income2Age.m | AgeData.m | Income2Education.m | EducationData.m

```
clear;
clc;
syms x;
syms y;
syms z;
%Education
eqn = exp(-(x-3.113)^2/(2*0.756^2))/(0.7417*x^-2.289*0.756*(2*pi)^(1/2));
g = diff(eqn);
disp(g)
x = 1:0.1:6;
b = subs(eqn);
plot(x,b)
title("Education");
S = solve(eqn,x);
disp(S);

%Age
eqn1 = 100*(1/(5.54*(2*pi)^(1/2)) * exp(-(y-31.167)^2/(2*5.54^2)))/(0.7806*exp(-((y+2.061)/42.14)^2) + 1.442*exp(-((y-40.69)/29.5)^2));
g = diff(eqn1);
disp(g)
y = 16:1:65;
b = subs(eqn1);
h = ones(50);
h = h(:,1);
figure
plot(y,b);
title("Age");
hold on;
plot(y,h);
hold off;
S = solve(eqn1,y);
disp(S)

%Income
eqn2 = 1.015/0.545*exp(-(log(z)-3.35429)^2/(2*0.545^2))/exp(-(log(z)-2.569)^2/(2*1.015^2));
g = diff(eqn2);
disp(g)
a = 95.873;
subs(a)
z = 10:1:180;
b = subs(eqn2);
figure
plot(z,b);
title("Income");
S = solve(eqn2,z);
disp(S)
```

B.3.3 Education Data Code

StanleyDataProcess.m | Income2Age.m | AgeData.m | Income2Education.m | EducationData.m

```
clear
clc;
%Pure Curve Fitting ?No Noise)
clear;
clc;
Education = zeros(0);
%Data Generation
%1
r = 0*rand(130,1) + 1;
r = reshape(r,1,length(r));
Education = [Education,r];
%2
r = 0*rand(1040,1) + 2;
r = reshape(r,1,length(r));
Education = [Education,r];
%3
r = 0*rand(7200,1) + 3;
r = reshape(r,1,length(r));
Education = [Education,r];
%4
r = 0*rand(1030,1) + 4;
r = reshape(r,1,length(r));
Education = [Education,r];
%5
r = 0*rand(400,1) + 5;
r = reshape(r,1,length(r));
Education = [Education,r];
%6
r = 0*rand(200,1) + 6;
r = reshape(r,1,length(r));
Education = [Education,r];
disp(length(Education))
%Graph Drawing
histogram(Education)
%Curve Fitting
histfit(Education,6,'normal')
%Find Graph Parameters
i = reshape(Education,10000,1);
pd = fitdist(i,'normal')
x = 1:1:6;
out = pdf(pd,x);
figure
plot(x,out)
```

B.3.4 Income to Education Restriction Code

StanleyDataProcess.m | Income2Age.m | AgeData.m | Income2Education.m

```
%X Value: Education lvl
%Y Value: Income
clear;
clc;
x = [1,2,3,4,5,6];
y = [27,37,40,43,72,90];
scatter(x,y)
syms f(a)
f(a) = 19.01*exp(0.2567*a);
y1 = f(x);
hold on;
plot(x,y1)
hold off;
```

B.3.5 Income to Age Restriction Code

StanleyDataProcess.m | Income2Age.m | AgeData.m

```
%X Value: Age
%Y Value: Income Kilo-USD
clear
clc;
x = [17.5,22,29.5,39.5,49.5,59.5,65];
y = [24,30,42,52,52,52,49];
y = y/10*7;
[p,S] = polyfit(x,y,2);
disp('R^2')
answer = 1 - (S.normr/norm(y - mean(y)))^2
disp('Equation')
disp(p)
x1 = 16:0.1:70;
y1 = polyval(p,x1);
scatter(x,y)
hold on
plot(x1,y1)
hold off
```

B.3.6 Demographic Factors Model

StanleyDataProcess.m | AgeData.m

```
clear;
clc;
%Age Fixing
A = importdata("AgeData.txt");
X =A(:,1);
Y = A(:,2);
Z = A(:,3);
%Percentage = Z(YearsOld + 1);

%Income Fixing
%1
Income = zeros(0);
r = 2.86*rand(400,1) + 0;
r = reshape(r,1,length(r));
Income = [Income,r];
%2
r = 4.28*rand(1500,1) + 2.86;
r = reshape(r,1,length(r));
Income = [Income,r];
%3
r = 4.29*rand(3500,1) + 7.14;
r = reshape(r,1,length(r));
Income = [Income,r];
%4
r = 5.71*rand(1500,1) + 11.43;
r = reshape(r,1,length(r));
Income = [Income,r];
%5
r = 11.43*rand(1300,1) + 17.14;
r = reshape(r,1,length(r));
Income = [Income,r];
%6
r = 42.86*rand(1200,1) + 28.57;
r = reshape(r,1,length(r));
Income = [Income,r];
%7
r = 71.43*rand(500,1) + 71.43;
r = reshape(r,1,length(r));
Income = [Income,r];
%8
r = 10*rand(100,1) + 142.86;
r = reshape(r,1,length(r));
Income = [Income,r];
%Graph Drawing
histogram(Income,50)
%Fitting
```

B.3.7 Processing Age Code

AgeData.m

```
%Pure Curve Fitting ?No Noise)
clear;
clc;
Age = zeros(0);

%Data Generation
%1
r = 6*rand(785,1) + 18;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%2
r = 4*rand(2985,1) + 25;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%3
r = 4*rand(3820,1) + 30;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%4
r = 4*rand(1785,1) + 35;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%5
r = 5*rand(480,1) + 40;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%6
r = 4*rand(145,1) + 46;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];

%Graph Drawing
histogram(Age)

%Curve Fitting
histfit(Age,10,'normal')

%Finding Graph Parameters
i = reshape(Age,10000,1);
pd = fitdist(i,'normal')
```

AgeData.m


```
r = 4*rand(2985,1) + 25;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%3
r = 4*rand(3820,1) + 30;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%4
r = 4*rand(1785,1) + 35;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%5
r = 5*rand(480,1) + 40;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];
%6
r = 4*rand(145,1) + 46;
r = round(r);
r = reshape(r,1,length(r));
Age = [Age,r];

%Graph Drawing
histogram(Age)

%Curve Fitting
histfit(Age,10,'normal')

%Finding Graph Parameters
i = reshape(Age,10000,1);
pd = fitdist(i,'normal')
```


```
x = 20:1:50;
out = pdf(pd,x);
figure
plot(x,out)

%Function Creation
A = importdata("AgeData.txt");
X =A(:,1);
Y = A(:,2);
Z = A(:,3);
```